Structure of ^{8}B and astrophysical S_{17} factor

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Abstract. Nuclear structure data are of crucial importance in order to address important astrophysical problems such as the origin of chemical elements, the inner working of our Sun, and the evolution of stars. We demonstrate this by investigating the ground state structure of ^{8}B and ^{7}Be nuclei within the Skyrme Hartree-Fock framework and by calculating the overlap integral of ^{8}B and ^{7}Be wave functions. The latter is used to calculate the astrophysical S factor (S_{17}) for the solar fusion reaction ^{7}Be$(p,\gamma)^8$B.

1. Introduction

Nuclear astrophysics research addresses some of the most fundamental questions in nature *e.g.,* the origin of the elements that make up our bodies and our world, and formation and evolution of the sun, the stars and the galaxies. There is an intimate connection between nuclear physics inputs and studies of these fascinating astrophysical phenomena [1]. A diverse set of nuclear data is required to model the composition changes and energy release in astrophysical environments ranging from Big Bang to inner working of our own Sun to exploding stars. Theoretical studies as well as measurements of microscopic nuclear physics processes provide the foundation for models to understand various astrophysics phenomena. These models are now challenged by incredibly detailed observations from ground based and space based (*e.g.,* CHANDRA X-ray Observatory, Hubble Space Telescope) observational devices that give us an unprecedented view of the Cosmos

We shall demonstrate the role of the nuclear structure physics in nuclear astrophysics by considering the case of the Solar fusion reaction ^{7}Be(p,γ)^{8}B, which is of great importance to the solar neutrino issue and to other related astrophysical studies. ^{8}B is the source of the high energy neutrinos from the Sun that are detected in the SNO, Kamiokande and Homestake experiments [2, 3, 4]. It is, therefore, a crucial requirement to determine as accurately as possible the cross section of this reaction at relative energies corresponding to solar temperatures (about 20 keV). In this energy region, the cross-section $\sigma_{p\gamma}(E_{cm})$ [which is usually expressed in terms of the astrophysical $S_{17}(E_{cm})$ factor] of the ^{7}Be(p,γ)^{8}B reaction is directly proportional to the high energy solar neutrino flux. A better knowledge of S_{17} is, therefore, important to improve the precision of the theoretical prediction of ^{8}B neutrino flux from present and future solar neutrino experiments.

$S_{17}(0)$] is determined either by direct measurements [5, 6] or by indirect methods such as Coulomb dissociation [7] and transfer reactions [8, 9]. Efforts have also been made to calculate the cross section of this reaction within the framework of the shell model and the cluster model [10, 11]. The key point of these calculations is the determination of the wave functions of ^{8}B states within the given structure theory.

We have investigated the structure of ^{8}B in the framework of the Skyrme Hartree-Fock (SkHF) model which has been used successfully to describe the ground-state properties of both stable [12, 13] as well as exotic nuclei [14]. The SkHF method with density-dependent pairing correlation and SLy4 interaction parameters has been successful in reproducing the binding energies and rms radii [14] in the light neutron halo nuclei ^{6}He, ^{8}He, ^{11}Li and ^{14}Be. We solve spherically symmetric Hartree-Fock (HF) equations with SLy4 [15] Skyrme interaction which has been constructed by fitting to the experimental data on radii and binding energies of symmetric and neutron-rich nuclei. Pairing correlations among nucleons have been treated within the BCS pairing method. We have, however, renormalized the parameter of the spin-orbit term of the SLy4 interaction so as to reproduce the experimental binding energy of the last proton in the ^{8}B nucleus [16]. A check on our interaction parameters was made by calculating binding energies and rms radii of ^{7}Be, ^{7}B, ^{8}Li and ^{9}C nuclei with the same set where a good agreement is obtained with corresponding experimental data. The overlap integral of the HF wave functions for ^{7}Be and ^{8}B ground states has been used to calculate the astrophysical S_{17} factor.

2. Results and discussions

2.1. Structure calculations

The values of various parameters of the $SLy4$ Skyrme effective interaction as used in our calculations are given in [16]. The rms radii for matter (r_m), neutron (r_n) and proton (r_p) distributions are presented in Table I for five light nuclei. Also shown in this table are the matter, neutron, and proton radii (under the column "Expt") extracted by methods in which measured reaction (or interaction) cross sections are fitted by theoretical models having them as input parameters. The quantities listed under "Theory" column are the results of our calculations. Here r_m is obtained by summing the average of proton and neutron radii in every orbit weighted with occupation probabilities. We see that for all the isotopes the calculated r_m is in good agreement with the corresponding values listed under the "Expt" column.

2.2. Valence proton radius in ^8B and Astrophysical S_{17} factor

We define a overlap function of the bound state wave functions of two nuclei B and A, where $B = A + p$ (p represents a proton) as

$$I_A^B(\mathbf{r}) \quad = \quad \int d\xi \Psi^*_{AI_AM_A}(\xi)\Psi_{BI_BM_B}(\mathbf{r},\sigma_p,\xi), \qquad (1)$$

Table 1: Rms mass (r_m), proton (r_p) and neutron (r_n) radii for various nuclei. Theoretical results obtained with modified SLy4 Skyrme force. Under the column "Expt", are the values of corresponding radii extracted by fitting the reaction or interaction cross sections by different theoretical methods as discussed in the text. We have defined $r_i =< r_i^2 >^{1/2}$

Nucl.	rms radii(fm)					
	"Expt"			Theory		
	r_m	r_p	r_n	r_m	r_p	r_n
^7Be	2.33 ± 0.02	-	-	2.49	2.63	2.29
^7B	-	-	-	2.86	3.18	1.84
^8Li	2.37 ± 0.02	2.26 ± 0.02	2.44 ± 0.02	2.54	2.29	2.67
^8B	2.55 ± 0.08	2.76 ± 0.08	2.16 ± 0.08	2.57	2.73	2.27
	2.43 ± 0.03	2.49 ± 0.03	2.33 ± 0.03			
^9C	2.42 ± 0.03			2.59	2.77	2.20

where I_A and I_B are the total spins of nuclei A and B, respectively, and $\mathbf{r}$ is the position of the proton with respect to the c.m. of nucleus A. σ_p is the spin variable of the proton, and ξ stands for the remaining set of internal variables which also include isospins. In this expression nuclear wave functions Ψ_A and Ψ_B are supposed to be properly translational invariant. The Hartree-Fock wave functions calculated in the previous section may not be so despite the fact that a c.m. correction factor has been incorporated in the energy functional. Corrections for the spurious c.m. motion should, therefore, be applied to the HF wave functions before using them in Eq. (1). However, effects of such corrections on the one-particle overlap function calculated within the shell model have been found [17, 18] to be of the order of only about 2-5 %. It is quite likely that the situation will be no different for the HF case.

We, now, present results for the astrophysical S_{17} factor calculated using HF overlap functions. In the region outside the core where range of the nuclear interaction becomes negligible, the radial overlap wave function of the bound state can be written as

$$R^B_{A\ell j}(r) \quad \simeq \quad \bar{c_{lj}} W_{\eta,l+1/2}(2kr)/r, \tag{2}$$

where W is the Whittaker function, k the wave number corresponding to the single proton separation energy and η the Sommerfield parameter for the bound state. In Eq. (2), $\bar{c_{lj}}$ is the asymptotic normalization constant, required to normalize the radial overlap wave function to the Whittaker function in the asymptotic region. The S_{17} factor is related to the proton capture cross section as

$$S_{17}(E) = \sigma(E) E \, e^{(2\pi\eta(E))}. \tag{3}$$

At the zero energy the S_{17} factor depends only on $\bar{c_{lj}}$ and one can write [10, 11]

$$S_{17}(0) = \kappa \sum_j \bar{c}_{1j}^2, \tag{4}$$

In Ref. [10], κ (= 37.8) has been obtained by using a microscopic cluster model for the scattering states, while a value of 36.5 has been reported for this quantity in Ref. [11] using a hard sphere scattering state model. However, once the relevant integration distances are sufficiently enhanced in [11] the value of κ there comes out to be 37.2, which is in good agreement with that of the microscopic model.

In our calculations, we have used our HF overlap function directly in the calculation of the amplitudes for the direct capture reaction. The amplitude for the radiative capture reaction can be written as $b + c \rightarrow a + \gamma$ which can be written as

$$M_{fi} \;\; = \;\; \langle \phi_a(\xi_b, \xi_c, r) | \hat{O}(r) | \phi_b(\xi_b) \phi_c(\xi_c) \psi_{k_i}^{(+)}(\mathbf{r}) \rangle, \tag{5}$$

where ϕ, ξ and $\mathbf{r}$ are the wave function, and the internal coordinate of the bound particle and the relative coordinate between b and c, respectively. $\hat{O}(r)$ is the electromagnetic operator. $\psi_{k_i}^{(+)}(\mathbf{r})$ is the distorted wave in the initial $b+c$ channel. The overlap of initial and final bound state wave functions is defined by

$$\begin{aligned} I_{bc}^a(\mathbf{r}) \;\; &= \;\; \langle \phi_a(\xi_b, \xi_c, r) | \phi_b(\xi_b) \phi_c(\xi_c) \rangle, \\ &= \;\; \sum_{\ell j \mu} < I_b M_b j m | I_a M_a >< \ell m - \mu s \mu | j m > i^\ell R_{b\ell j}^a(r) Y_{\ell m - \mu}(\hat{r}) \\ &\times \;\; \chi_{s\mu}, \end{aligned} \tag{6}$$

It may be noted that at low relative energies the distortion effects are goverened solely by Coulomb interactions. Substituting Eq. (6) into Eq. (5), expanding $\psi_{k_i}^{(+)}(\mathbf{r})$ in terms of the partial waves and carrying out the angular momentum algebra, the amplitude M_{fi} can be written in terms of the wave function $R_{b\ell j}^a(r)$ and the radial coulomb wave functions for the initial channel. Therefore, the overlap functions $R_{b\ell j}^a(r)$ calculated in the HF theory can be used to calculate the capture cross sections at low relative energies without any uncertainty of other input quantity. It may further be noted that using Eq. (2), we can also calculate the asymptotic normalization constants $\bar{c}_{\ell j}$.

In Table II, we show the results for the asymptotic normalization constants for the $p_{1/2}$ and $p_{3/2}$ proton orbits and the the astrophysical S-factor obtained by this method. Our $\bar{c}_{3/2}$ agrees almost perfectly with the value of this quantity determined experimentaly in Ref. [19]. However, our $\bar{c}_{1/2}$ is larger than that reported by these authors by a factor of about 1.5. They have determined these quantities (by invoking mirror symmetry) from the ANCs of the ^{8}Li $\rightarrow$ ^{7}Li $+n$ system extracted from the measurement of the neutron transfer reaction ^{13}C(^{7}Li,^{8}Li)^{12}C. This difference in the calculated and "measured" values of $\bar{c}_{1/2}$ is significant as a larger $\bar{c}_{1/2}$ leads to a bigger S_{17} as compared to that reported in Ref. [19]. There may be a need to relook in the theoretical analysis of the data reported in Ref. [19]. In the DWBA

calculations of the ^{13}C(^{7}Li,^{8}Li)^{12}C reaction, same set of potentials have been used for ^{7}Li + C and ^{8}Li + C channels. However, ^{8}Li is an unstable system; thus optical potential in the final channel could be different from that of the incoming (^{7}Li) channel. It is also not very clear from Ref. [19] if the spin-orbit terms have been used in the optical potentials and bound state potentials. This could affect the $p_{1/2}$ and $p_{3/2}$ components differently. More work is clearly needed to clarify this issue.

Table 2: SkHF results for asymptotic normalization coefficients ($\bar{c}_{\ell j}$), and astrophysical $S-$ factors $S^A_{17}(0)$ and $S^B_{17}(0)$ and $S^C_{17}(0)$. S^A_{17} corresponds to results obtained by using the HF overlap function directly to a capture code while S^B_{17} and S^C_{17} to those obtained by using Eq. (4) with $\kappa = 36.5$ and 37.8, respectively.

Observable	Our Result
$\bar{c}_{13/2}$	0.64
$\bar{c}_{11/2}$	0.34
$S^A_{17}(0)(eVb)$	22.0
$S^B_{17}(0)(eVb)$	19.5
$S^C_{17}(0)(eVb)$	20.2

The recent ^{7}Be(p,γ) measurements yield values of $S_{17}(0)$ which are clustered around 18.5 eV b [5] and 22.0 eV b [6]. Our $S_{17}(0)$ (22.0 eV barn) is closer to two latest direct capture measurement results which claim good accuracy. However, this is larger than the most recent result reported by the Texas A & M group from the ANC method [19]. This is also slightly larger than the values obtained from the Coulomb dissociation (CD) method [20], even thought it has been argued [21] that there is no significant difference between the CD and diect capture values of $S_{17}(0)$.

3. Summary and conclusions

In summary, in this paper we studied the structure of ^{8}B, and ^{7}Be nuclei within the Skyrme Hartree-Fock (SkHF) framework. We calculated binding energies, various densities distribution and rms radii for these nuclei. Using the same set of the force parameters, we obtain good agreements with experimental values of binding energies and rms matter radii for all these nuclei. We have calculated the overlap function $<^7\mathrm{Be}|^8\mathrm{B}>$ from the SkHF wave functions which has been employed to extract the asymptotic normalization coefficients for the $^8\mathrm{B} \rightarrow ^7\mathrm{Be} + p$ system. We obtain an astrophysical S-factor of 22.0 eV b which lies within the adopted limits ($19.1^{+4.0}_{-1.0}$ eV b) of the near zero energy astrophysical S-factor. Our work clearly

shows that proper nuclear structure input is vital in order to understand some very important issues in nuclear astrophysics.

4. Acknowledgment

This work is supported by the Department of Atomic Energy, (BRNS), (BARC), Mumbai, under contract no. 2001/37/14/BRNS/699.

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